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A THERMAL FRONT CIRCULATION MODEL

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THERMAL FRONT CIRCULATION MODEL

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ABSTRACT. A model of the thermal bar circulation in a large lake, of small aspect ratio is presented. Unlike models that use prescribed values of the eddy viscosity and diffusivity, this model utilizes a second order turbulence-closure scheme to compute flow variables and turbulence properties as part of the overall solution.

1. INTRODUCTION

In his study of the thermal structure of Lake Ladoga, Tikhomirov (1963) suggested that the thermal bar is a common feature of all large lakes in temperate regions during spring and autumn. Tikhomirov described the thermal bar as a moving boundary between “the thermo-passive and thermo-active regions” of the lake, and also recognized that the thermal bar influences limnological, biological, optical, and chemical properties of the lake and, therefore, affects “aspects of life of a lake.”

Boyce (1989) has reviewed studies on the thermal structure of the Great Lakes. This work presents the historical development of physical limnology, the seasonal thermal structure, and general circulation in the Great Lakes.

Rodgers (1966) measured the surface and cross section temperatures of Lake Ontario in the spring of 1965 and the winter of 1965-66 at times when the lake had isotherms above and below 4^{o}C. Drogue measurements near the thermal bar showed convergence at the bar of warmer, shallower, stratified water and of the deeper and colder mid-lake water. Rodgers also recorded the migration to deeper water of the thermal bar as a function of time, depth, bathymetry, and temperature gradients.

A theoretical study of the thermal circulation in Lake Michigan was carried out by Huang (1971) in 1971 in which the velocity and temperature fields were obtained by solving the equations of motion and using a perturbation and matched asymptotic expansion scheme. The model was steady and neglected any along-front variation. It appeared to predict the mean circulation pattern in the lake with reasonable accuracy.

A generalization of Huang’s steady-state model was carried out in 1971 by Bennett (1971) in a numerical study. In Bennett’s calculations, the model basin was taken as an infinitely long rotating channel with no along-channel variation. The equations of motion were reduced to a system of prediction equations which were solved for the temperature, the along-channel velocity, and the stream function at each time step. A motion was predicted that is confined to the stratified region where a geostrophic current parallel to shore is dominant. Also found were a zone of sinking water centered around the 4^{o}C isotherm and a zone of upwelling in the shallow regions.

A combined laboratory and theoretical investigation of the thermal bar was carried out by Elliott (1971). In the theoretical part of the study, Elliott neglected horizontal advection and the diffusion of heat, and computed the temperature field using a one-dimensional heat diffusion equation. Two expressions for the temperature distribution were derived, one for the deeper, convective, unstable side of the bar and the other for the shallower, stratified, stable side. The velocity field was determined using the vorticity equation under the assumption that the flow is driven by buoyancy and viscous forces, and after replacing the temperature field by an approximate expression valid on both sides of the bar. In light of the reasonably good agreement of the mathematical model with laboratory measurements, Elliott extended the model to natural lakes, even though it did not incorporate rotation effects.

All of the theoretical studies reviewed above use a constant eddy viscosity and diffusivity model, or some combination of spatially variable eddy viscosity and diffusivity coefficients. None of them used a turbulence model.

Difficulties in obtaining accurate values of the viscosity and diffusivity coefficients for various flow situations justify looking for alternative models. Turbulence models have been developed to handle successfully various flows. Even though the complexity of such models makes a problem difficult to solve, they have the advantage of computing the turbulence characteristics as part of the solution to the flow, and generally do a much better job than constant coefficients models. For this reason, the present study uses a second order turbulence model to describe the mean flow.

2. GOVERNING EQUATIONS

Using the Boussinesq and f-plane approximations, the equations of motion describing viscous, heat-conducting flow on the rotating earth are

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + f\varepsilon_{ik3} u_i = -\frac{1}{\rho_o}\frac{\partial \rho}{\partial x_i} + v\nabla^2 u_i - g\frac{\rho}{\rho_o}\delta_{i3} \tag{1}$$

$$\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \Gamma\nabla^2 T \tag{2}$$

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{3}$$

$$\rho = P(T) \tag{4}$$

Here, we have used Einstein's tensorial notation, with summation on the repeated indices, f is the Coriolis parameter, v and Γ are the molecular kinematic viscosity and thermal diffusivity, T is the temperature, ρ is a temperature dependent equation of state in polynomial form as used by Mamayev (1975). g is the acceleration of gravity, and δ_{ij}, is the Kronecker delta. The conditions at the upper and lower boundaries of the fluid will be specified later. For large scale turbulent geophysical flows, effects due to molecular viscosity and diffusivity can be neglected except in viscous sublayers near solid boundaries, which will not be treated here. Hence, the terms involving v and Γ in (1) and (2) will be omitted throughout our work.

3. DONALDSON'S TURBULENCE CLOSURE MODEL

We employ Reynolds' approach of averaging the equations of motion as follows. A generic dependent variable θ will be expressed as the sum Θ + θ', where Θ and θ' denote the mean and fluctuating components, and where an overbar denotes the Reynolds average. The averaged continuity, momentum, and energy equations are

$$\frac{\partial U_i}{\partial x_i} = 0 \tag{5}$$

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} + f\varepsilon_{ik3} U_i = \frac{1}{\rho_0}\frac{\partial p}{\partial x_i} - g\frac{\rho - \rho_0}{\rho_0}\delta_{i3} - \frac{\partial \overline{u_i' u_j'}}{\partial x_i} \tag{6}$$

$$\frac{\partial T}{\partial t} + U_i \frac{\partial T}{\partial x_i} = \frac{\partial \overline{u_i' T'}}{\partial x_i} \qquad (7)$$

Here, and for the remainder of this work, T denotes the mean-flow temperature.Subscripts *i*, *j*, and *k* take the values 1, 2, and 3. The presence of the Reynolds stresses, $\overline{u_i' u_j'}$, and the temperature flux, $u_i' T'$, make the system of equations incomplete. Lewellen (1977) presents a model for the correlations of the fluctuating variables devised by Donaldsont, which has been extensively verified for and successfully applied to various engineering and geophysical flows. The equations of Donaldson's model are:

$$\frac{D\tau_{ij}}{Dt} + \tau_{ik}\frac{\partial U_j}{\partial x_k} + \tau_{ik} f\left(\varepsilon_{ki3}\tau_{ki} + \varepsilon_{kj3}\tau_{ki}\right) = c_i \frac{\partial}{\partial x_k}\left(q\Lambda \frac{\partial \tau_{ij}}{\partial x_k}\right) - \frac{q}{\Lambda}\left(\tau_{ij} + \delta_{ij}\frac{q^2}{4}\right) - \alpha g\left(\delta_{i3}h_j + \delta_{j3}h_i\right) \qquad (8)$$

$$\frac{Dq^2}{Dt} + 2\tau_{ik}\frac{\partial U_i}{\partial x_k} + c_1 \frac{\partial}{\partial x_k}\left(q\Lambda \frac{\partial q^2}{\partial x_k}\right) - \frac{q^3}{4\Lambda} + 2\alpha g h_3 \qquad (9)$$

$$\frac{D\Lambda}{Dt} + c3\frac{\Lambda}{q^2}\tau_{ik}\frac{\partial U_i}{\partial x_k} = c_1 \frac{\partial}{\partial x_k}\left(q\Lambda \frac{\partial \Lambda}{\partial x_k}\right) + c_2 q - \frac{c_4}{q}\frac{\partial (q\Lambda)}{\partial x_k}\frac{\partial (q\Lambda)}{\partial x_k} + \frac{c_8\Lambda}{q^2}\alpha g h_3 \qquad (10)$$

$$\frac{Dh_i}{Dt} + f\varepsilon_{ki3}h_k = \tau_{ik}\frac{\partial T}{\partial x_k} - h\frac{\partial U_i}{\partial x_k} + \alpha g\varphi\delta_{i3} + c_1 \frac{\partial}{\partial x_k}\left(q\Lambda \frac{\partial h_i}{\partial x_k}\right) - c_6\frac{q}{\Lambda}h_i \qquad (11)$$

$$\frac{D\varphi}{Dt} = -2h_k\frac{\partial T}{\partial x_k} + c_1 \frac{\partial}{\partial x_k}\left(q\Lambda \frac{\partial \varphi}{\partial x_k}\right) - c_7\frac{q}{\Lambda}\varphi \qquad (12)$$

Here we have defined $\tau_{ij} = \overline{u'_i\, u'_j}$ as the Reynolds stress tensor divided by the density, $h_i = \overline{u'_i\, T'}$ is the turbulent heat flux in *i* direction, $\varphi = \left(T'\right)^2$ as the mean square temperature fluctuation, $q = \sqrt{\overline{u_i' u_i'}}$ as twice the turbulent kinetic energy, Λ as the macroscale length of turbulence, and $D/Dt = \partial/\partial t + U_i\left(\partial/\partial x_i\right)$ as the total derivative operator. The quantities c_i are constants given in Table 1 and are slightly different from those given in Lewellen (1977), which uses an inappropriate value of 0.36 for the Karman constant ***k*.**

Table 1. Constants of Donaldson's Turbulence Model for k=0.4

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.3	0.07769	0.35	0.375	0.672717	0.75	0.45	0.8

4. BOUNDARY CONDITIONS

We neglect water waves and assume that the flows near the water surface and the bottom satisfy the usual logarithmic law of the wall. Let $\vec{S}$ denote the tangential component of the stress at a boundary, $\overrightarrow{N}$ the normal to the boundary, and $N = \pm\left|\vec{N}\right|$ where *N* is positive or negative depending on whether the fluid lies above or below the boundary. Then, according to the model, the following holds as $N \to 0$:

$$\vec{\tau} \to \left|\vec{S}\right|\left(\vec{N}\vec{S} + \vec{S}\vec{N}\right) - 2^{1/2}\left(\left|\vec{S}\right|^2 \vec{I} + \vec{S}\vec{S}\right) \tag{13}$$

$$q \to 25^{5/4}\left|\vec{S}\right| \tag{14}$$

$$\vec{h} \to Q\left\{\overrightarrow{N} - \frac{1+c_6}{2^{1/2}}\frac{\vec{S}}{\left|\vec{S}\right|}\right\} \tag{15}$$

$$\varphi \to 2^{1/2}\,\frac{c_6}{c_7}\,\frac{Q^2}{\left|\vec{S}\right|^2} \tag{16}$$

Here μ = *sgn(N)* is the sign of *N*, z_σ, is the roughness length of the boundary, *Q* is the temperature flux, the heat flux divided by $\rho_0\, C_p$, $\vec{I}$ is the identity matrix, $\overrightarrow{U}_\sigma$ is the velocity at the boundary, and T_σ, is the temperature at the boundary σ. At the air-sea interface, $\vec{S}$ is the bottom stress and z_σ, is the sea surface roughness length, while at the bottom, $\vec{S}$ represents the bottom friction and z_σ the bottom roughness length.

5. SCALING OF THE EQUATIONS

Let (x_1, x_2) denote the horizontal axes, and x_3 the vertical axis, and let Greek subscripts take on the values 1 or 2. Also let *L* denote a characteristic horizontal length scale, *D* a vertical length scale, *U* a characteristic horizontal velocity, U_τ a characteristic friction velocity, T_0 a reference temperature, and Q_0 a characteristic value of the surface temperature flux *Q*. The time it takes a particle with velocity U_τ to travel through distance ***D*** is the vertical penetration time $t_0 = D/U_\tau$. To evaluate *U* and $\Delta T \equiv T - T_0$, we balance the time derivative and the vertical derivative of the Reynolds stress in the horizontal momentum equations, and the time derivative and the vertical derivative of the temperature flux in the energy equation, taking t_0 as a characteristic time scale, $(U\tau)^2$ as a characteristic scale for $\overline{u_i^{'}u_j^{'}}$ and Q_0, as a characteristic scale for $\overline{u_i^{'}T^{'}}$. This yields $U = U_\tau$ and $\Delta T = Q_{0/}U_\tau$. Thus, we scale the variables using the transformations

$$\left(\vec{S}, q\right) = U_\tau\left(\overrightarrow{S^*}, q^*\right), \qquad Q = Q_0 Q^* \tag{20}$$

$$x_\alpha = L x_\alpha{}^*, \qquad \left(x_3, H, \Lambda\right) = D\left(x_3{}^*, H^*, \Lambda^*\right) \tag{21}$$

$$f = f_0 f^*, \qquad t = \frac{D}{U_\tau} t^* \tag{22}$$

$$U_\alpha = U_\tau U_\alpha{}^*, \qquad U_3 = U_\tau \frac{D}{L} U_3{}^* \tag{23}$$

$$T = T_0 + \frac{Q_0}{U_\tau} T^*, \qquad \tau_{ij} = U_\tau^2 \tau_{ij}{}^* \tag{24}$$

$$\varepsilon_T = \frac{k}{\ln \frac{D}{z_T}}, \qquad \varepsilon_B = \frac{k}{\ln \frac{D}{z_B}}, \tag{25}$$

$$h_i = Q_o h_i^*, \qquad \varphi = \frac{Q_o^2}{U_\tau^2} \varphi^* \tag{26}$$

where asterisks denote dimensionless variables, quantities ε_T and ε_B serve as the square roots of characteristic top- and bottom-drag coefficients, and H is the bottom profile function.

6. APPROXIMATION FOR NATURAL BASINS

Using the scaling (20) to (26), we arrive at the dimensionless governing equations of the thermal bar circulation in a lake in temperate regions. No analytical solution of these equations is unknown, and their numerical solution is tedious. Approximations are needed because computational cost and efficiency are of real concern. The main approximation to be used here is based on the restriction on the aspect ratio, $\delta \equiv D/L << 1$, which is satisfied for most naturally occurring bodies of water. As a result, the following approximated equations will be valid for any body of water in temperate regions with a small. We define the functions and constants

$$\beta + \frac{2\Omega}{R_0} \cos\varphi_0, \qquad \lambda = \frac{\beta L}{f_0} \tag{27}$$

$$f = 1 + \lambda y, \qquad r = \frac{U_\tau}{f_0 D} \tag{28}$$

where Ω is the angular rotation of the earth and R_0, its radius, φ_0 is the latitude angle, λ is the non-dimensionalized β, and *r* is the vertical Rossby number. The scaled and approximated continuity, and momentum equations become, with asterisks omitted,

$$\frac{\partial U_\gamma}{\partial x_\gamma} + \frac{\partial U_3}{\partial x 3} = 0 \tag{29}$$

$$r \frac{\partial U_\alpha}{\partial t} + f \varepsilon_{\gamma\alpha 3} U_\gamma + \frac{\partial p}{\partial x_\alpha} = r \frac{\partial \tau_{\alpha 3}}{\partial x_3} \tag{30}$$

$$\frac{\partial p}{\partial x_3} = B(T) \tag{31}$$

$$\frac{\partial T}{\partial t} = -\frac{\partial h_3}{\partial x_3} \tag{32}$$

The scaled approximated turbulence model equations become

$$\frac{\partial \tau_{13}}{\partial t} + \tau_{33}\frac{\partial U_1}{\partial x_3} - \frac{f}{r}\tau_{23} = c_1\frac{\partial}{\partial x_3}\left(q\Lambda\frac{\partial \tau_{13}}{\partial x_3}\right) - \frac{q}{\Lambda}\tau_{13} - R(T)h_1 \tag{33}$$

$$\frac{\partial \tau_{23}}{\partial \tau} + \tau_{33}\frac{\partial U_2}{\partial x_3} + \frac{f}{r}\tau_3 = c_1\frac{\partial}{\partial x_3}\left(q\Lambda\frac{\partial \tau_{23}}{\partial x_3}\right) - \frac{q}{\Lambda}\tau_{23} - R(T)h_2 \tag{34}$$

$$\frac{\partial \tau_{33}}{\partial \tau} = c_1\frac{\partial}{\partial x_3}\left(q\Lambda\frac{\partial \tau_{23}}{\partial x_3}\right) - \frac{q}{\Lambda}\left(\tau_{33} + \frac{q^2}{4}\right) - 2R(T)h3 \tag{35}$$

$$\frac{\partial q^2}{\partial \tau} = 2\left\{\tau_{13}\frac{\partial U_1}{\partial x_3} + \tau_{23}\frac{\partial U_2}{\partial x_3}\right\} + c_1\frac{\partial}{\partial x_3}\left(q\Lambda\frac{\partial \tau^2}{\partial x_3}\right) - \frac{q^3}{4\Lambda} + 2R(T)h_3 \tag{36}$$

$$\frac{\partial \Lambda}{\partial t} + \frac{c_3\Lambda}{q^2}\left\{\tau_{13}\frac{\partial U_1}{\partial x_3} + \tau_{23}\frac{\partial U_2}{\partial x_3}\right\} = c_1\frac{\partial}{\partial x_3}\left(q\Lambda\frac{\partial \Lambda}{\partial x_3}\right) + c_2 q - \frac{c_4}{q}\left[\frac{\partial (q\Lambda)^2}{\partial x_3}\right] + c_8 R(T)\frac{\Lambda}{q^2}h_3 \tag{37}$$

$$\frac{\partial h_1}{\partial t} - \frac{f}{r}h_2 = \tau_{13}\frac{\partial T}{\partial x_3} - h_3\frac{\partial U_1}{\partial x_3} + c_1\frac{\partial}{\partial x_3}\left(q\Lambda\frac{\partial h_1}{\partial x_3}\right) - c_6\frac{q}{\Lambda}h_1 \tag{38}$$

$$\frac{\partial h_2}{\partial t} + \frac{f}{r}h_1 = \tau_{23}\frac{\partial T}{\partial x_3} - h_3\frac{\partial U_2}{\partial x_3} + c_1\frac{\partial}{\partial x_3}\left(q\Lambda\frac{\partial h_2}{\partial x_3}\right) - c_6\frac{q}{\Lambda}h_2 \tag{39}$$

$$\frac{\partial h_3}{\partial t} = \tau_{33}\frac{\partial T}{\partial x_3} + R(t)\varphi + c_1\frac{\partial}{\partial x_3}\left(q\Lambda\frac{\partial h_3}{\partial x_3}\right) - c_6\frac{q}{\Lambda}h_3 \tag{40}$$

$$\frac{\partial \varphi}{\partial t} = 2h_3\frac{\partial T}{\partial x_3} + c_1\frac{\partial}{\partial x_3}\left(q\Lambda\frac{\partial \varphi}{\partial x_3}\right) - c_7\frac{q}{\Lambda}\varphi \tag{41}$$

Here, *R(T)* is a temperature-dependent thermal expansion function, and *B(T)* is the buoyancy per unit mass. They are polynomial functions of *T*. At the surface boundary x_3=0, we denote T^s as the surace temperature, and $\vec{S}$ the surface wind stress. Using the scaling (20) to (26), the upper boundary conditions become

$$U_\alpha \to U_\alpha^s - s_\alpha \left(\frac{1}{\varepsilon_T} + \frac{1}{k} \ln|x_3| \right) \tag{42}$$

$$U_3 = 0 \tag{43}$$

$$T \to T^s + c_6 \frac{Q}{\left|\vec{S}\right|} \left(\frac{1}{\varepsilon_T} + \frac{1}{k} \ln|x_3| \right) \tag{44}$$

$$T_{\alpha 3} = \left|\vec{S}\right| S_\alpha \tag{45}$$

$$\tau_{33} = 2^{1/2} \left|\vec{S}\right|^2 \tag{46}$$

$$q \to 2^{5/4} \left|\vec{S}\right| \tag{47}$$

$$\Lambda \to c_5 |x_3| \tag{48}$$

$$h_\alpha \to Q \frac{(1 + c_6)}{2^{1/2} c_6} \left(\frac{s_\alpha}{\left|\vec{S}\right|} \right) \tag{49}$$

$$h_3 \to Q) \tag{50}$$

$$\varphi \to 2^{1/2} \frac{c_6}{c_7} \frac{Q^2}{\left|\vec{S}\right|^2} \tag{51}$$

At the bottom boundary, $x_3 == H(x_\alpha)$ we donote T^B as the bottom temperature, and $\vec{b}$ the bottom stress. The scaled lower boundary conditions reduce to

$$U_\alpha \to b_\alpha \left(\frac{1}{\varepsilon_\beta} + \frac{1}{k} \ln|x_3 + H| \right) \tag{52}$$

$$U_3 + U_\alpha \frac{\partial H}{\partial x_\alpha} \to 0 \tag{53}$$

$$T = T^\beta \tag{54}$$

$$\tau_{\alpha 3} = \left|\vec{b}\right| b_\alpha \tag{55}$$

$$\tau_{33} = -2^{1/2} \left|\vec{b}\right|^2 \tag{56}$$

$$q \to 2^{5/4} \left| \vec{b} \right| \tag{57}$$

$$\Lambda \to c_5 \left| x_3 + H \right| \tag{58}$$

$$h_i = \varphi = 0 \tag{59}$$

To complete the formulation of the model, a surface heat balance must be carried out. The surface conditions involve a temperature flux term *Q*, an external forcing applied by the atmosphere. The evaluation of the heat flux exchanged through the atmosphere-air interface is done using radiative transfer equations of the type used by McCormick (1987) and Ivanoff (1972). Cloud and fog effects may be included if more precision is sought and/or data are available. In any case, an accurate parameterization of the surface temperature flux is critical to any accurate description of the thermal bar. The surface currents U_1^s, U_2^s and temperature T^s are evaluated using equations (42) and (45) respectively.

7. CONCLUSION

The system of equations (29) to (41) along with the boundary conditions (42) to (59) is closed and constitutes the mathematical description of the thermal bar. This model uses a secondorder turbulence closure scheme described in Lewellen (1977), and holds for most naturally occurring bodies of water of small aspect ratio and located in temperate regions. The equations can be solved numerically for the flow variables and the turbulence characteristics. Numerical results have been obtained and will be presented elsewhere.

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9. REFERENCES

Bennett, J.R. Thermally driven lake currents during the spring and fall transition periods. Proceedings, 14th Conference on Great Lakes Research, International Association for Great Lakes Research, pp. 535-544 (1971).

Boyce, EM., M.A. Donelan, P.F. Hamblin, CR. Murthy, and T.J. Simmons. Thermal structure and circulation in the Great Lakes. *Atmosphere-Ocean 27:607-642* (1989).

Elliott, G.H. A mathematical study of the thermal bar. Proceedings, 14th Conference on Great Lakes Research, International Association for Great Lakes Research, pp. 545-554 (1971).

Huang, J.C.K. The thermal current in Lake Michigan. *Journal of Physical Oceanography* 1:105-122 (1971).

Ivanoff, A. Oceanic absorption of solar energy. In *Modeling and Prediction of the Upper Layers of the Ocean.* E. B. Kraus (ed). Pergamon Press, Oxford, pp. 47-71 (1972).

Lewellen, W.S. *Handbook of Turbulence.* Walter Frost and Trevor H. Moulden (eds). Plenum Press, New York, pp. 237-280 (1977).

Li Ting, Y. Formation of Thermocline in Great Lakes. Proceedings, 13th Conference on Great Lakes Research, pp. 453-467. International Association for Great Lakes Research (1970).

Mamayev, O.I. *The Temperature-Salinity Analysis of the World Ocean Waters.* Elsevier Oceanographic Series, Vol. 11. Elsevier Publishing Company. Amsterdam, pp. 9-47 (1975).

McCormick, M.J. Mixed layer modeling of tideless seas. Ph.D. Dissertation, The University of Michigan, Ann Arbor (1987).

Rodgers, G.K. The thermal bar in Lake Ontario, Spring 1965 and Winter 1965-66. Pub. No. 15, Great Lakes Research Division, The University of Michigan, Ann Arbor (1966).

Tikhomirov, A.I. The thermal bar of Lake Ladoga. *Bulletin (Izvestia) All-Union Geog. Sot.*, 95:134-142. American Geophysical Union, Soviet Hydro.:Selected Paper, No. 2 (1963).